CHAPTER - 8

INTRODUCTION TO TRIGONOMETRY

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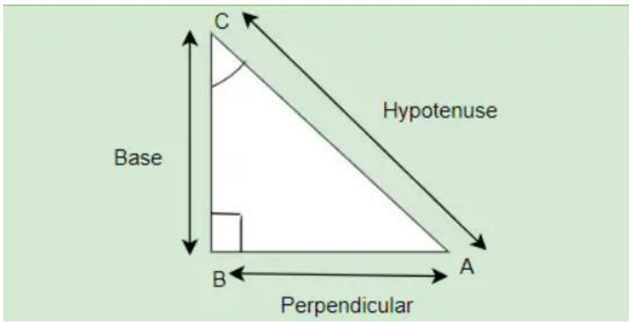
TRIGONOMETRY: A branch of mathematics that deals with the relationships between the angles and sides of triangles. It has numerous applications in various fields, including physics, engineering, and architecture.

TRIGONOMETRIC RATIOS: The ratios of the sides of a right triangle with respect to its acute angles, called trigonometric ratios of the angle.



For acute angle C, side opposite to C is perpendicular, adjacent side BC is base and side opposite to angle B is hypotenuse

so Trigonometric ratios of an acute angle C in a right angle triangle are as follows

FOR ANGLE C	FOR ANGLE A
sin C=Perpendicular/Hypotenuse=AB/AC cosec C= Hypotenuse/Perpendicular=AC/AB cos C=Base/Hypotenuse=BC/AC sec C=Hypotenuse/Base=AC/BC tan C=Perpendicular/Base=AB/BC cot C=Base/Perpendicular=BC/AB	sin A=Perpendicular/Hypotenuse=BC/AC cosec A= Hypotenuse/Perpendicular=AC/BC cos A=Base/Hypotenuse=AB/AC sec A=Hypotenuse/Base=AC/AB tan A=Perpendicular/Base=BC/AB cot A=Base/Perpendicular=AB/BC

EXAMPLE 1: In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) sin A, cos A

(ii) sin C, cos C

Solution:

In a given triangle ABC, right angled at B = ∠B = 90°

Given: AB = 24 cm and BC = 7 cm

According to the Pythagoras Theorem,

In a right- angled triangle, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides.

By applying Pythagoras theorem, we get

$AC^2=AB^2+BC^2$

$AC^2 = (24)^2+7^2$

$AC^2 = (576+49)$

$AC^2 = 625cm^2$

$AC = \sqrt{625} = 25$

Therefore, AC = 25 cm

(i) To find Sin A, Cos A

We know that sine (or) Sin function is the equal to the ratio of length of the opposite side to the hypotenuse side. So it becomes

Sin A = Perpendicular /Hypotenuse = BC/AC = 7/25

Cosine or Cos function is equal to the ratio of the length of the adjacent side to the hypotenuse side and it becomes,

Cos A = Base /Hypotenuse = AB/AC = 24/25

Relation between Trigonometric Ratios:

- **Cosec A=1/sinA**
- **secA = 1/cosA**
- **tan A = sinA/cosA**
- **cotA = cosA/sinA=1/tanA**

EXAMPLE 2: Given sec θ = 13/12 Calculate all other trigonometric ratios

Solution:

We know that sec function is the reciprocal of the cos function which is equal to the ratio of the length of the hypotenuse side to the adjacent side

Let us assume a right angled triangle ABC, right angled at B

sec θ =13/12 = Hypotenuse/Adjacent side = AC/AB

Let AC be 13k and AB will be 12k

Where, k is a positive real number.

According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides of a right angle triangle and we get,

$AC^2=AB^2 + BC^2$

Substitute the value of AB and AC

$(13k)^2= (12k)^2 + BC^2$

$169k^2= 144k^2 + BC^2$

$169k^2= 144k^2 + BC^2$

$BC^{2\ =}\ 169k^2 – 144k^2$

$BC^2= 25k^2$

Therefore, BC = 5k

Now, substitute the corresponding values in all other trigonometric ratios

So,

Sin θ = Opposite Side/Hypotenuse = BC/AC = 5/13

Cos θ = Adjacent Side/Hypotenuse = AB/AC = 12/13

tan θ = Opposite Side/Adjacent Side = BC/AB = 5/12

Cosec θ = Hypotenuse/Opposite Side = AC/BC = 13/5

cot θ = Adjacent Side/Opposite Side = AB/BC = 12/5

TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES:

θ	0^0	30^0	45^0	60^0	90^0
sinθ	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	**1**
cosθ	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	**0**
tanθ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	**Not defined**
cotθ	**Not defined**	$\sqrt{3}$	**1**	$\frac{1}{\sqrt{3}}$	**0**
secθ	**1**	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	**2**	**Not defined**
cosecθ	**Not defined**	**2**	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	**1**

Example 3:

Evaluate: sin 60° cos 30° + sin 30° cos 60°

Solution:

sin 60° cos 30° + sin 30° cos 60°

First, find the values of the given trigonometric ratios

sin 30° = 1/2

cos 30° = √3/2

sin 60° = √3/2

cos 60°= 1/2

Now, substitute the values in the given problem

sin 60° cos 30° + sin 30° cos 60° = √3/2 ×√3/2 + (1/2) ×(1/2) = 3/4+1/4 = 4/4 =1

Example 4:

Evaluate:$2tan^2 45^0 + cos^2 30^0 - sin^2 60^0$

Solution: As $tan45^0=1$, $cos30^0=\frac{\sqrt{3}}{2}$, $sin60^0=\frac{\sqrt{3}}{2}$

Therefore putting the values in given expression;

$=2(1)^2+(\frac{\sqrt{3}}{2})^2-(\frac{\sqrt{3}}{2})^2$

= 2+0=2

Trigonometric Identities: An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle:

- **$sin^2A+cos^2A=1$** or $sin^2A=1- cos^2A$ or $cos^2A=1- sin^2A$
- **$1+tan^2A=sec^2A$**or $sec^2A- tan^2A =1$ or $tan^2A = sec^2A -1$
- **$1+cot^2A=cosec^2A$** or $cosec^2A- cot^2A=1$ or $cot^2A= cosec^2A-1$

EXAMPLE 5: **EVALUATE: $9 sec^2A – 9 tan^2A$**

SOLUTION:Take 9 outside, and it becomes

$9 sec^2A – 9 tan^2A$

$= 9 (sec^2A – tan^2A)$

$= 9×1 = 9$ $(\because Sec^2A - tan^2A = 1)$

Therefore, 9 $\sec^2 A - 9\tan^2 A = 9$

EXAMPLE 6:

Prove that: $(\text{cosec}\ \theta - \cot\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$

Proof: To prove this, first take the Left-Hand side (L.H.S) of the given equation, to prove the Right Hand Side (R.H.S)

L.H.S. = $(\text{cosec}\ \theta - \cot\theta)^2$

The above equation is in the form of $(a-b)^2$, and expand it

Since $(a-b)^2 = a^2 + b^2 - 2ab$

Here a = cosec θ and b = cot θ

$= (\text{cosec}^2\theta + \cot^2\theta - 2\text{cosec}\ \theta\cot\theta)$

Now, apply the corresponding inverse functions and equivalent ratios to simplify

$= (1/\sin^2\theta + \cos^2\theta/\sin^2\theta - 2\cos\theta/\sin^2\theta)$

$= (1 + \cos^2\theta - 2\cos\theta)/(1 - \cos^2\theta)$

$= (1-\cos\theta)^2/(1 - \cos\theta)(1+\cos\theta)$

$= (1-\cos\theta)/(1+\cos\theta)$ = R.H.S.

Therefore, $(\text{cosec}\ \theta - \cot\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$

Hence proved.

PRACTICE QUESTIONS SECTION-A

MULTIPLE CHOICE QUESTIONS:

1. sin 2B = 2 sin B is true when B is equal to

(a) 90° (b) 60° (c) 30° (d) 0°

*2.The value of cos 0°. cos 1°. cos 2°. cos 3°… cos 89° cos 90° is

(a) 1 (b) -1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

3.What is the minimum value of sin A, $0 \le A \le 90°$

(a) -1 (b) 0 (c) 1 (d) 1/2

4. If θ is an acute angle of a right angled triangle ,then which of the following equation is not true?

(a) $sin\theta cot\theta = \cos\theta$ (b) $cos\theta tan\theta = sin\theta$

(c) $cosec^2\theta - cot^2\theta = 1$ (d) $tan^2\theta - sec^2\theta = 1$

*5. If $\sin\theta + \sin^2\theta = 1$, then $\cos^2\theta + \cos^4\theta =$

(a) -1 (b) 0 (c) 1 (d) 2

**6. 5 $\tan^2 A - 5 \sec^2 A + 1$ is equal to

(a) 6 (b) -5 (c) 1 (d) -4

7. If sin A = 1/2 and cos B = 1/2, then A + B = ?

(a) 90^0 (b) 30^0 (c) 60^0 (d) 0^0

**8. If 4tanA=3, then $\frac{4sinA-cosA}{4sinA+cosA}$

(a) 2/3 (b) 1/3 (c) 1/2 (d) ¾

9.(sin30° + cos30°) – (sin 60° + cos60°)

(a) – 1 (b) 0 (c) 1 (d) 2

10. Value of tan30°/cot60° is:

(a) 1/√2 (b) 1/√3 (c) 1 (d) √3

**11. If x tan 45° sin 30° = cos 30° tan 30°, then x is equal to

(a) √3 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

***12. If sec A + tan A = x, then tan A =

(a) $\frac{x^2-1}{x}$ (b) $\frac{x^2-1}{2x}$ (c) $\frac{x^2+1}{x}$ (d) $\frac{x^2+1}{2x}$

13. $\frac{1-cosA}{sinA}$ is equal to

(a) $\frac{sinA}{1-cosA}$ (b) $\frac{sinA}{1+cosA}$ (c) $\frac{cosA}{1-cosA}$ (d) $\frac{cosA}{1+cosA}$

**14. If sin A – cos A = 0, then the value of $\sin^4 A + \cos^4 A$ is

(a) 2 (b) 1 (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

15. If in ΔABC, ∠C = 90°, then sin (A + B) =

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

**16. 9 $\sec^2 A - 9 \tan^2 A$ =

(a) 1 (b) 9 (c) 8 (d) 0

17. (1 + tan θ + sec θ) (1 + cot θ – cosec θ)

(a) 0 (b) 1 (c) 2 (d) – 1

18. (sec A + tan A) (1 – sin A) =

(a) sec A (b) sin A (c) cosec A (d) cos A

*19. $1+\tan^2 A/1+\cot^2 A$ =

(a) $\sec^2 A$ (b) -1 (c) $\cot^2 A$ (d) $\tan^2 A$

**20.2tan 30°/1+tan^{2}30° =

(a) sin 60° (b) cos 60° (c) tan 60° (d) sin 30°

SECTION-B

**21.Evaluate: $5cosec^2 45^0 - 3sin^2 90^0 + 5cos0^0$

22.If tan$\theta = \frac{3}{4}$, then find the value of $cos^2\theta - sin^2\theta$.

23.Find the value of $\frac{cos30^0+sin60^0}{1+cos60^0+sin30^0}$.

24.Given 15cotA=8,findsinA and secA.

*25.Evaluate: $sin60^0 cos30^0 + sin30^0 cos60^0$

**26.If cot$\theta = \frac{7}{8}$,Evaluate: $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$.

*27. If sinx+cosy =1,and x=30^0 and y is an acute angle, find the value of y.

28. If sin$\theta = x\ and\ sec\theta = y\ , find\ cot\theta.$

29. Express cosA in terms of cotA.

**30. If sinA=cosA, find $2tan^2A + sin^2A - 1$.

*31. If $\sqrt{3}sin\theta - cos\theta = 0\ and\ 0 < \theta < 90, find\ the\ value\ of\ \theta.$

***32. If $sec\theta + tan\theta = p$ then find the value of cosecθ.

33.In $\Delta ABC\ right\ angled\ at\ C\ and\ AC = \sqrt{3}$BC, then prove that: $\measuredangle ABC = 60^0$.

*34. If (1+cosA)(1-cosA)=3/4,find secA.

*35. If cosecA=5/3,Find cosA+tanA

SECTION-C

***36 .Prove that:$\sqrt{sec^2\theta + cosec^2\theta}$ = tan$\theta + cot\theta$

**37. Prove that $\frac{cotA-cosA}{cotA+cosA} = \frac{cos^2A}{(1+sinA)^2}$

38. Prove that (sec$\theta + tan\theta)(1 - sin\theta) = cos\theta$

*39. Prove that $\frac{1+secA}{secA} = \frac{sin^2A}{1-cosA}$

**40. If cos$\theta + sin\theta = \sqrt{2}cos\theta, show\ that\ cos\theta - sin\theta = \sqrt{2}\ sin\theta$

41. Prove that $\frac{1+cotA-cosecA}{1+tanA+secA} = 2$

*42. Prove that $\frac{tan\theta}{1-tan\theta} - \frac{cot\theta}{1-cot\theta} = \frac{cos\theta+sin\theta}{cos\theta-sin\theta}$

43. If sin$\theta = \frac{12}{13}$,Find $\frac{sin^2\theta-cos^2\theta}{2sin\theta cos\theta} \times \frac{1}{tan^2\theta}$

***44. Prove that $\frac{sin^3\theta+cos^3\theta}{sin\theta+cos\theta} = 1 - sin\theta cos\theta$

**45. Prove that $\sqrt{\frac{1+cos\theta}{1-cos\theta}} = cosec\theta + cot\theta$

SECTION-D

*46. If 7sin^{2}A+3cos^{2}A=4,then show that tanA= $\frac{1}{\sqrt{3}}$

*47. Evaluate:$\frac{4cot^2 60^0+sec^2 30^0-2sin^2 45^0}{sin^2 60^0+cos^2 45^0}$

**48. If sin θ + cos θ = √3, then prove that tan θ + cot θ = 1.

**49. Prove that:$\frac{tan\theta}{1+cot\theta} + \frac{cot\theta}{1-tan\theta} = 1 + sec\theta cosec\theta$

**50. Prove that:$\sqrt{\frac{sec\theta-1}{sec\theta+1}} + \sqrt{\frac{sec\theta+1}{sec\theta-1}} = 2cosec\theta$

**51. Prove that:$(\sin\theta + \sec\theta)^2 + (\cos\theta + \text{cosec}\,\theta)^2 = (1 + \sec\theta\,\text{cosec}\,\theta)^2$

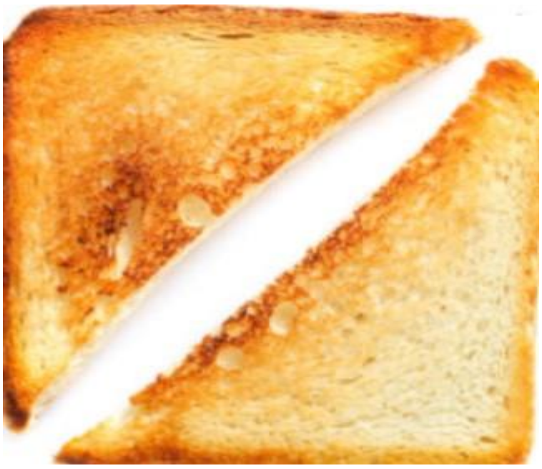
*52. Prove that: $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta) = 13$.

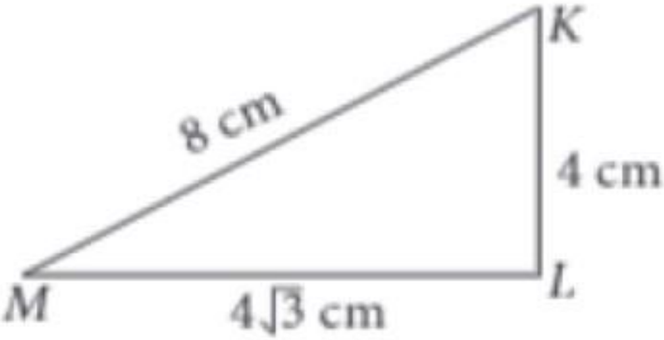
*53. Prove that:sin A(1 + tan A) + cos A(1 + cot A) = sec A + cosec A

***54. If sec θ – tan θ = x, show that: sec θ=1/2(x+1/x) and tan θ=1/2(1/x-x)

CASE STUDY QUESTIONS

55. Astha is feeling so hungry and she thought to eat something. She looked into the fridge and found a bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found it forms a right-angled triangle, with sides 4 cm, 4√3 cm and 8 cm.





On the basis of above information, answer the following questions.

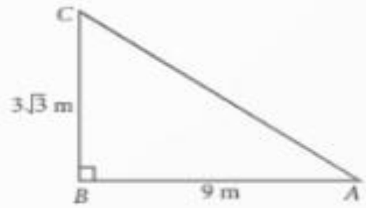
(i) The value of ∠M is : A. 30° B. 60° C. 45° D. None of these

(ii) The value of ∠K is : A. 45° B. 30° C. 60° D. None of these

(iii) Find the value of tan*M:* A. √3 B. 1/√3 C. 1 D. None of these

(iv)$\sec^2 M - 1 =$?: A. tanM B. $tan^2 M$ C. tan2M D. None of these

56. Three friends - Om, Nitish and Sahil are playing hide and seek in a park. Om and Nitish hide in the shrubs and Sahil have to find both of them. If the positions of three friends are at A, Band C respectively as shown in the figure and forms a right angled triangle such that AB = 9 m, BC = 3√3 m and ∠B = 90°, then answer the following questions.



(i) The measure of $\angle A$ is :

(a) 30^0 (b) 45^0 (c) 60^0 (d) none of these

(ii) The measure of $\angle C$ is :

(a) 30^0 (b) 45^0 (c) 60^0 (d)none of these

(iii) The length of AC is:

(a) $\sqrt{3}$ m (b) $2\sqrt{3}\ m$ (c) $4\sqrt{3}$ m (d) $6\sqrt{3}$ m

(iv) cos2A=

(a)0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d)$\frac{\sqrt{3}}{2}$

(v) $\sin\frac{C}{2}$ =

(a)0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d)$\frac{\sqrt{3}}{2}$

Answer key CHAPTER-8 SOLUTIONS:

1.d , 2.c , 3.b , 4.d , 5.d , 6.d ,7.a ,8.c ,9.b ,10.c ,11.d ,12.b , 13.b , 14.d ,15.d ,16.b ,17.c 18.d ,19.d ,20.a , 21.12 , 22.7/25 ,23.$\frac{\sqrt{3}}{2}$,24. sinA=15/17,secA=17/8 ,25.1 ,26. 49/64 ,27.y = 60^0 ,28. 1/xy ,29.$cosA=\frac{cotA}{\sqrt{1+cot^2A}}$,30.3/2 ,31.30^0 ,32.$cosec\theta=\frac{p^2+1}{p^2-1}$,34. 2 ,35.31/20 ,43. 595/3456 , 47. 4/3 ,55.(i) a (ii)c (iii)b (iv) c ,56.(i)a (ii)c (iii)d (iv)b (v)d